Non-local Model for the Spatial Distribution of Impact Ionization Events in Avalanche Photodiodes

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Abstract

We report an extension of the analytical Dead Space Multiplication Theory [IEEE Trans. Electr. Dev., vol. 39, pp. 546–552, 1992] that provides the means to analytically determine the spatial distribution of electron and hole impact-ionization events in an arbitrarily specified heterojunction multiplication region. The model can be used to understand the role of dead space in regularizing the locations of impact ionization. It can also be utilized to analyze, design and optimize new generations of ultra-low noise, multi-staged gain avalanche photodiodes based upon judiciously energizing and relaxing carriers to enhance electron impact ionizations and suppress hole impact ionizations.

1 Introduction

Avalanche photodiodes (APDs) are widely deployed in high-data-rate optical-fiber communication and laser radar systems that operate at the wavelengths of 1.3 and μm. Among the APD structures, the separate absorption, charge and multiplication (SACM) InP-InGaAs APDs have been the preferred structure for two reasons. First, they have high sensitivity, which results from their internal carrier multiplication, namely the avalanche of impact ionizations that result from each photogenerated carrier. Second, they are highly cost effective compared to receivers that employ optical pre-amplification. However, due to the stochastic nature of the impact ionization process the multiplication gain comes at the expense of extra noise. This multiplication noise is characterized by a quantity termed the excess noise factor, which accounts for the gain uncertainty.

Various approaches have been explored to reduce the excess noise factor of APDs. They include the use of thin multiplication regions and impact-ionization engineered (I^2E) multiplication regions. Both of these approaches exploit the dead-space effect to reduce the excess noise by making the spatial distribution of impact ionizations more deterministic [1, 2]. The dead space is the minimum distance a carrier must travel before it gains sufficient energy from the electric field to cause an impact ionization. Another approach is to suppress the impact ionization of holes (or electrons), $\beta \to 0$ (or $\alpha \to 0$), to make α and β as dissimilar as possible. According to the local field theory, both the gain-bandwidth product and the excess noise of APDs improve when one of the ionization coefficients is much larger than the other [3, 4]. As such, there has been a growing interest in APD structures that suppress the impact ionization of holes (or electrons) by impact-ionization engineering of the multiplication region [5, 6]. In these structures the relaxation of one type of carrier (to prevent it from impact ionizing) is achieved by judiciously engineering the different layers of the heterojunction multiplication region and the electric field profile therein. A key factor in the successful design of this multi-layer multiplication regions is the ability to accurately determine the places at which electrons and holes trigger impact ionization events.

In this letter we report an extension of the analytical *Dead Space Multiplication Theory* (DSMT) [7] that enables determining the spatial distribution of the impact-ionization events within an arbitrarily specified heterojunction multiplication regions. In fact, the newly developed recursive equations allow us to determine the number of electron and hole impact-ionization events individually within any sub-region of the multiplication region. Moreover, the model can accommodate carrier relaxation, which can be used to suppress the impact ionizations of one type of carriers.

2 Model

Consider an arbitrary multiplication region extending from $x = 0$ to $x = w$. We term an impact ionization event that is effected by a conduction-band electron an *electron-ionization event*; on the other hand, an impact ionization event that is effected by a valence-band hole is termed a *hole-ionization event*. Let A be any subset of the interval $[0, w]$. We are interested in computing the mean of the total number of electron-ionization events as well as hole-ionization events occurring in the subset A after a single parent carrier (at a prescribed location) initiates the avalanche process. If we can solve this problem for any subset A, then we can specialize it to the intervals $A_1 = [0, w/n), A_2 = [w/n, 2w/n), \ldots, A_n = [(n-1)w/n, w]$, and obtain the distribution of electron- and hole-ionization events throughout the multiplication region. (The partition parameter n is selected to achieve a desired spatial distribution resolution.) To solve this problem, we define $Z_e(x)$ and $Z_h(x)$ as the total stochastic number of electron impact-ionization events and hole impact-ionization events, respectively, when the avalanche process is triggered by a parent electron at location x. Similarly, we define $Y_e(x)$ and $Y_h(x)$ as the total stochastic number of electron impact-ionization events and hole impact-ionization events, respectively, when the avalanche process is triggered by a parent hole at location x. Before proceeding with the formulation of recursive equations that enable us to solve for the ensemble averages of the quantities $Z_e(x)$ and $Z_h(x)$, $Y_e(x)$ and $Y_h(x)$, let us formally introduce the probability density function (pdf) of the distance from the birth location of a carrier to the location of its first impact ionization thereafter. Following the notation in [2], let $h_e(\xi|x)$ denote the pdf of the distance, ξ, to the first ionization measured from the electron's birth position at x. Similarly, $h_h(\xi|x)$ denotes the pdf of the distance traveled by a hole born at x before it first ionizes. In the DSMT, $h_e(\xi|x)$ and $h_h(\xi|x)$ are described by the shifted-exponential models

$$h_e(\xi|x) = \begin{cases} \alpha(x+\xi)\exp\left(-\int_{d_e(x)}^{\xi} \alpha(x+y)\,dy \right), & \text{if } \xi \geq d_e(x) \\ \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

and

$$h_h(\xi|x) = \begin{cases} \beta(x-\xi)\exp\left(-\int_{d_h(x)}^{\xi}\beta(x-y)\,dy\right) & \text{if } \xi \geq d_h(x) \\ \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where $d_e(x)$ and $d_h(x)$ are the electron and hole dead spaces, respectively. The exact formulas for calculating the dead space can be found elsewhere [2].

We now invoke a renewal argument, similar to that introduced in [8] to obtain recursive (integral) equations for the mean of the quantities $Z_e(x)$ and $Z_h(x)$, $Y_e(x)$ and $Y_h(x)$, which we shall denote as $z_e(x)$ and $z_h(x)$, $y_e(x)$ and $y_h(x)$, respectively. Consider a parent electron at x initiating the avalanche process and suppose that its first ionization occurs at some location $\xi > x$. If we assume (for the moment) that $\xi \notin A$, then the conditional mean of $Z_e(x)$ given that the first ionization has occurred at ξ is simply $z_{e_1}(\xi) + z_{e_2}(\xi) + y_e(\xi)$, where $z_{e_1}(\xi)$ and $z_{e_2}(\xi)$ are the total average electron ionization events resulting from the two offspring electrons at ξ, while $y_e(\xi)$ is the total average electron ionization events resulting from the offspring hole at ξ. On the other hand, if the location ξ of the first ionization is in A, then upon the first ionization we will already have had one electron ionization, and this addition has to be accounted for. In this case, the conditional mean of $Z_e(x)$ is $1 + z_{e_1}(\xi) + z_{e_2}(\xi) + y_e(\xi)$. Of course, there is a chance that the parent electron does not impact ionize at all (with probability $\int_w^\infty h_e(\xi|x)\,d\xi$), in which case $Z_e(x)$ would be zero. By considering all of these scenarios while averaging over all possible locations ξ of the location of the first impact ionization (by the parent electron) and upon further simplification we obtain the integral equation

$$z_e(x) = \int_A h_e(\xi|x)\,d\xi + \int_x^w \Big[2z_e(\xi) + y_e(\xi)\Big] h_e(\xi|x)\,d\xi, \tag{3}$$

where the first term is simply the probability that the first ionization occurs in the region A. We can repeat the same argument to analyze the ensemble averages of the quantities $Z_h(x)$, $Y_e(x)$ and $Y_h(x)$; such analysis leads to three additional integral equations:

$$y_e(x) = \int_0^x \Big[2y_e(\xi) + z_e(\xi)\Big] h_h(\xi|x)\,d\xi \tag{4}$$

$$z_h(x) = \int_x^w \Big[2z_e(\xi) + y_e(\xi)\Big] h_e(\xi|x)\, d\xi \tag{5}$$

and

$$y_h(x) = \int_A h_h(\xi|x)\, d\xi + \int_0^x \Big[2y_h(\xi) + z_h(\xi)\Big] h_h(\xi|x)\, d\xi. \tag{6}$$

The coupled recursive equations (3), (4), (5) and (6) can be solved numerically using a simple iterative method.

3 Results

We have calculated the spatial distribution of electron- and hole-ionization events for two different cases of the multiplication region: (i) a hole-injection InP homojunction multiplication region, and (ii) an electron-injection heterojunction multiplication region. We have used a partition parameter, n, equal to 50 in the case of the homojunction multiplication region and 100 for the heterojunction multiplication region.

Figure 1 shows the calculated spatial distribution of the impact ionization events initiated by electrons and holes in an InP homojunction multiplication region of 150 nm under a constant electric field. It is assumed that parent holes are injected at $x = 0$. It can be seen from the figure that the number of hole-impact ionization events increases as the holes approach $x = 150$ nm while the number of electron impact ionizations events increases as the electrons approach $x = 0$. This is consistent with the fact that holes and electrons multiply as they acquire sufficient kinetic energy from the electric field traveling in opposite directions. More importantly, the figure shows the effect of the dead space on the spatial distribution of the impact ionization events. The distance at the beginning of the multiplication region, from $x = 0$ to $x \approx 37$ nm, in which holes do not impact ionize corresponds to the length of the hole dead space. In this portion of the multiplication region, holes have not gained sufficient energy to initiate an impact ionization event. Similarly, the distance at the end of the the multiplication region, from $x \approx 112$ nm to $x = 150$ nm, in which electrons do not impact ionize corresponds to the length of the electron dead space. As a comparison, Fig. 2 shows the calculated spatial distribution of the impact ionization events initiated by electrons and holes in the context of the local-field theory, i.e., neglecting the dead space. It is clear from Fig. 2 that the local-field theory does not capture the effect of the dead space on the localization of the impact ionization events. One important implication of this is that the local-field theory is unable to correctly predict the excess noise factor of thin multiplication regions (< 400 nm) where the dead space represents a significant portion of the multiplication region. Specifically, it is well know that the local-field theory overestimates the excess noise factor for thin APDs. For example, for the considered multiplication region operating at a mean gain of 15, the calculated excess noise factor predicted by the DSMT is around 5.4 while the local-field theory predicts an excess noise factor of around 10.5.

We have also calculated the spatial distribution of the electron- and hole-impact ionization events considering a single-carrier multiplication (SCM) APD with an InAlAs/InAlGaAs multiplication region. SCM-APDs were developed by Voxtel Inc. to obtain quasi-deterministic multiplication gains by suppressing hole-initiated impact ionization events [6, 9]. The multiplication region of an SCM-APD consists of a cascaded multiplier architecture, which combines various design techniques to suppress hole-initiated ionizations and enhance electron-initiated ionizations [6, 9]. Figure 3 shows the electric field profile across the multiplication region. The multiplication region has 5 multiplication cells, each of which consists of an avalanche layer, a hole relaxation layer, and an electron heating layer. The inset of Fig. 3 shows electric-field profile of the first two multiplications cells and the corresponding layers inside the cells. We assume that parent electrons are injected at $x = 0$. To model the mechanism of hole-relaxation we have adopted the approach described in [6, 9].

Lastly, Fig. 4 shows the distribution of the electron- and hole-ionization events across the multiplication region shown in Fig. 3. It can be seen from the figure that most of the impact ionization events occur in the lower bandgap and high ionization rate layers (InAlGaAs), which are the layers where the electric field is at a maximum. The figure also shows the large difference between the number of electron-impact ionizations compared to that of hole-impact ionizations. This disparity is a result of two factors: (1) the hole-relaxation layers, which prevent the holes form acquiring sufficient kinetic energy to impact ionize and subsequently reducing excess noise, and (2) the electrons are pre-heated prior injection into the InAlGaAs layer. These results are consistent with Voxtel's simulation results reported by Williams *et al.* [9].

4 Conclusion

We have developed an analytical recursive model to separately determine the distribution of electron-ionization events and hole-ionization events as the avalanche process is triggered by either a parent hole or electron at an arbitrary location inside multiplication region. The model can be used to simulate a mechanism for suppressing the impact ionizations triggered by one species of carrier and determine its effect on the localization of impact-ionization events. The new model is a crucial analytical tool for understanding, designing and optimizing new generations of APDs designed to achieve ultra-low noise characteristics by enhancing impact ionizations for electrons (holes) while suppressing impact the ionizations for holes (electrons).

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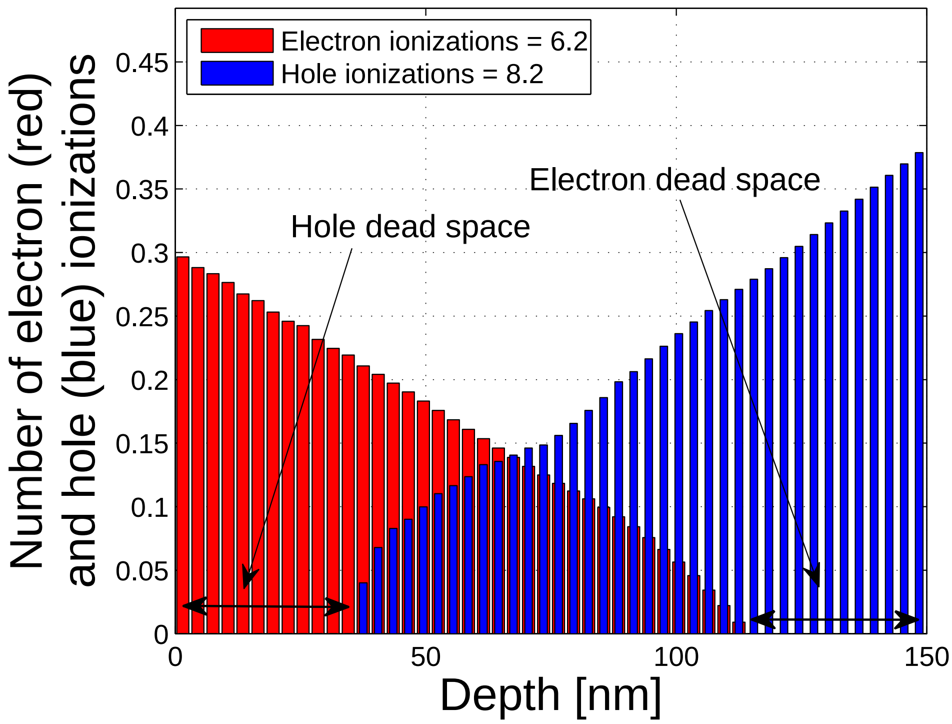

Figure 1: Spatial distribution of electron-impact ionization (red) and hole-impact ionization (blue) events for a InP homojunction multiplication region of 150 nm. The partition parameter, n, used in the calculations is 50.

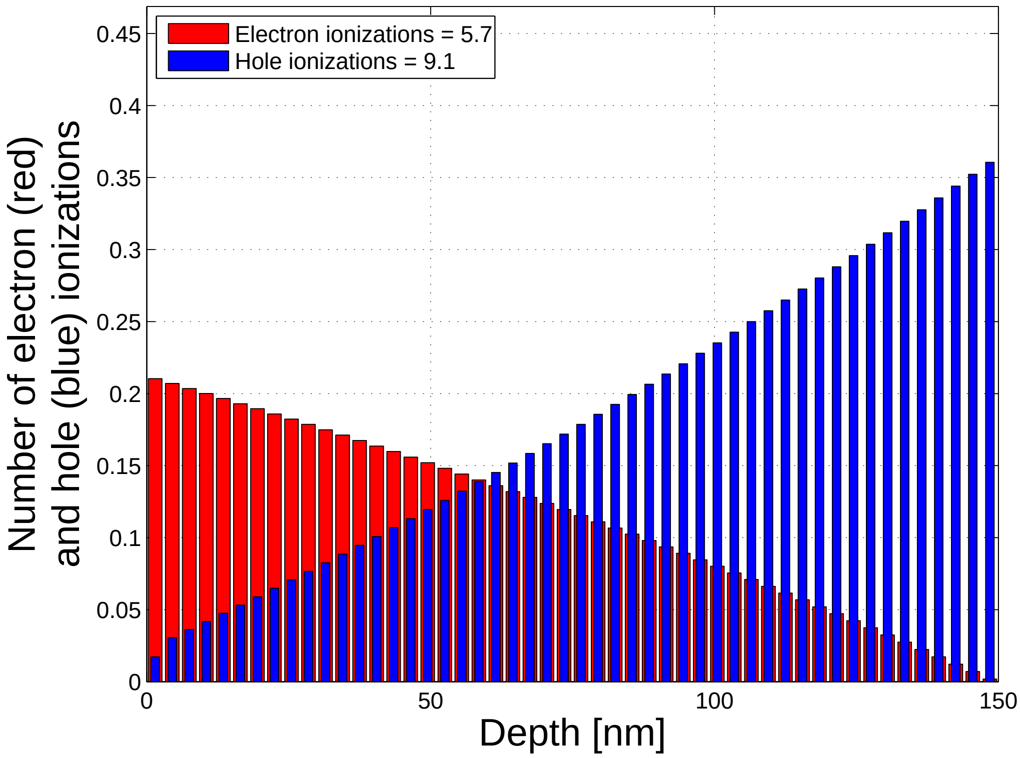

Figure 2: Spatial distribution of electron-impact ionization (red) and hole-impact ionization (blue) events for a InP homojunction multiplication region of 150 nm. This distribution was calculated in the context of the local-field theory, which neglects the hole and electron dead spaces. The partition parameter, n, is 50.

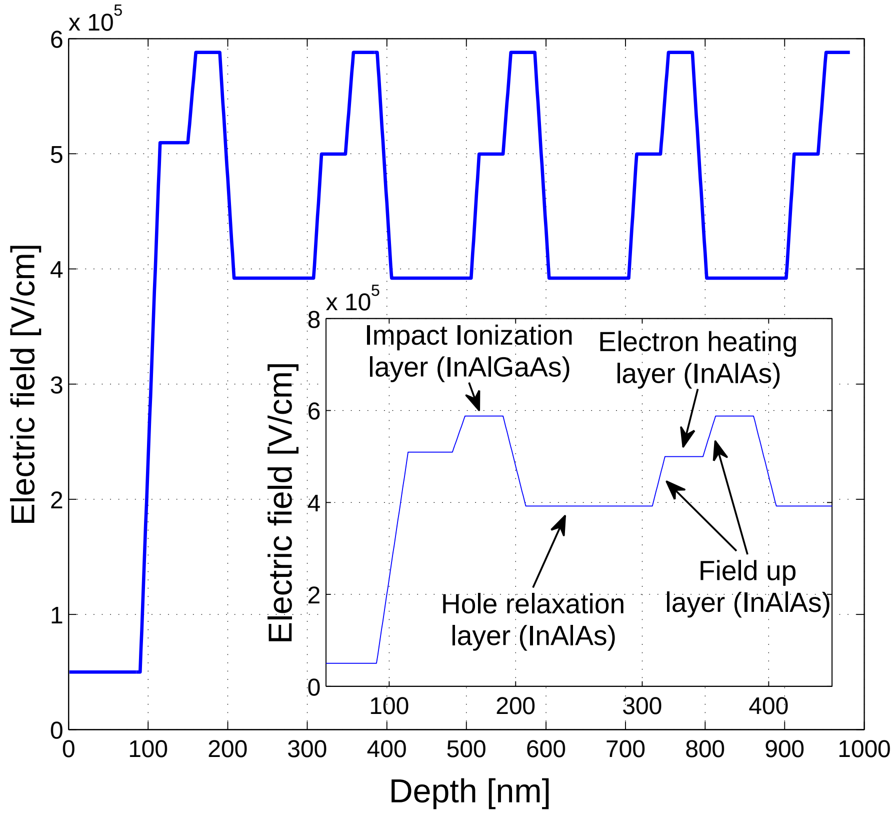

Figure 3: Electric-field profile across the InAlAs/InAlGaAs SCM APD multiplication region. The inset shows electric-field profile of the first two multiplications cells and the corresponding layers inside the cells. Details about the design of the multiplication cells can be found elsewhere [6, 9].

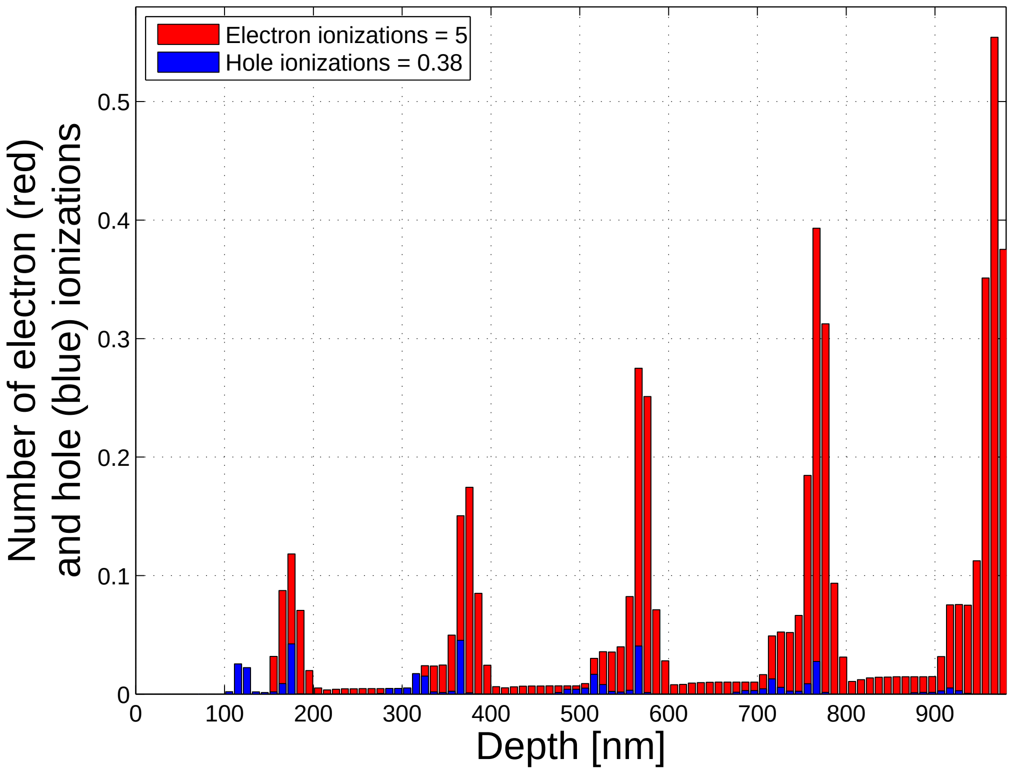

Figure 4: Spatial distribution of electron-impact ionization (red) and hole-impact ionization (blue) events for a SCM-APD with a multiplication region of 1 μm. The partition parameter, n, used in the calculations is 100.