Limitations of NEP as a Lidar APD Photoreceiver Performance Metric

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Abstract. Noise-equivalent power (NEP) is often used to express the voltage noise at the output of a photoreceiver amplifier in terms of an equivalent optical-signal level. However, avalanche photodiode (APD) receivers of the same NEP may differ substantially in terms of the amplitude distribution of the noise, resulting in substantial variation of the detection threshold required to extinguish false alarms in ladar or laser rangefinder APD receivers. Thus, NEP alone is not a good measure of the sensitivity of a laser-rangefinder receiver that employs an APD. The optical signal required to achieve a specified detection probability for a detection threshold that achieves a specified false-alarm rate is shown to be a more reliable characterization of APD photoreceiver performance for laser-ranging applications.

Keywords: ladar, lidar, avalanche photodiode, photoreceiver, receiver operating characteristic, pulse detection

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Introduction

Photoreceiver noise-equivalent power (NEP) can be an informative metric, but using NEP to evaluate laser-rangefinder (LRF) receiver performance comes with the limitation that the NEP of an APD receiver does not fully describe its receiver operating characteristic (ROC). The ROC of an LRF receiver is a parametric plot of the receiver's pulse-detection probability (Pd) for a specified signal level versus its false-positive rate, which can be determined from the receiver's false-alarm rate (FAR); both vary as functions of the detection threshold of the LRF's pulse-detection circuit. NEP uniquely characterizes the trend of FAR with threshold for non-APD receivers, but two APD receivers with the same NEP may have significantly different FAR characteristics.

Background

NEP and noise-equivalent input (NEI) are often used to express the voltage noise at the output of a photoreceiver's amplifier in terms of an equivalent optical-signal level that, if present at the detector's input, would result in an output-voltage swing of the same magnitude as the noise. Although NEP (W) and NEI (photons) can be defined in such a way as to quantify the signal level for which the signal-to-noise ratio is unity, these metrics are more typically used to quantify the noise present in dark conditions, without any signal shot noise.

A block diagram of an APD-based rangefinding photoreceiver is shown in Figure 1, where the output voltage noise (V_n) is the standard deviation of the potential at the transimpedance amplifier output (V_{out}) under dark conditions.

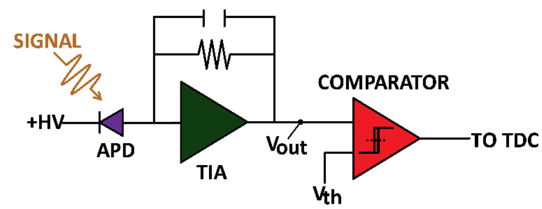

Figure 1. Block diagram of an APD-based rangefinding photoreceiver.

The deflection of V_{out} in response to a pulsed optical signal is written as either:

$$\Delta V_s = V_s - V_d = P_s\, QE\, M \frac{\lambda q}{hc}\, \Omega_{TIA}$$

$$= 0.8066\, \lambda\, QE\, M\, \Omega_{TIA}\, P_s \text{ [V]; or} \quad (1)$$

$$\Delta V_s = V_s - V_d = N_s\, QE\, M\, G_{TIA} \text{ [V]}, \quad (2)$$

where V_s is the output voltage when a pulsed optical signal is present, P_s is the peak instantaneous optical power of the signal in Watts, V_d is the output voltage when no signal is present, N_s is the total photon count of the signal pulse, QE is the quantum efficiency of the APD, M is the mean avalanche gain of the APD, λ is the signal wavelength in microns, q is the elementary charge in Coulombs, h is Planck's constant in J·s, c is the speed of light in m/s, Ω_{TIA} is the amps-to-volts transimpedance of the amplifier in Ω, and G_{TIA} is the electrons-to-volts conversion gain of the amplifier in V/e-.

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The noise-equivalent signals can then be written:

$$NEP = \frac{V_n}{0.8066\,\lambda\,QE\,M\,\Omega_{TIA}}\ \text{[W]; and} \tag{3}$$

$$\text{NEI} = \frac{V_n}{QE\,M\,G_{TIA}}\ \text{[photons]}. \tag{4}$$

In general, Ω_{TIA} and G_{TIA} depend on the signal pulse shape, so NEP and NEI are specific to the signals to which the dark noise is referred. Just as NEP refers the output-voltage noise to an instantaneous signal power and NEI refers the output-voltage noise to a total photon count, the output-voltage noise itself can be modeled as arising from either the APD’s instantaneous dark current or the cumulative charge delivered by the dark current over an effective integration time:

$$V_n = \Omega_{TIA}\sqrt{BW \times [S_{I\,TIA} + 2\,q\,M\,F(I_d + I_b)]}\ \text{[V]; or} \tag{5}$$

$$V_n = G_{TIA}\sqrt{noise_{TIA}^2 + \frac{\tau_{eff}}{q} M\,F\,(I_d + I_b)}\ \text{[V]}, \tag{6}$$

where BW is the bandwidth of the receiver’s analog signal chain into its comparator in Hz; $S_{I\,TIA}$ is the spectral intensity of the input-referred noise of the amplifier averaged over its bandwidth, expressed as:

$$S_{I\,TIA} = \left.\frac{V_n^2}{BW\,\Omega_{TIA}^2}\right|_{I_d, I_b = 0}\ \text{in A}^2\text{/Hz}; \tag{7}$$

F is the excess noise factor of an APD with ionization rate ratio k, expressed as $F = k\,M + (1 - k)\,(2 - 1/M)$; I_d is the multiplied DC dark current in amps; I_b is the background photocurrent in amps; $noise_{TIA}$ is the input-referred noise of the amplifier in units of electrons, expressed as:

$$noise_{TIA} = \left.\frac{V_n}{G_{TIA}}\right|_{I_d, I_b = 0}; \tag{8}$$

and τ_{eff} is effective integration time in s.

In most LRF receiver use cases, the pulse-detection threshold is adjusted to extinguish false alarms until a permissible FAR is achieved (such as 60 Hz); then, LRF sensitivity is expressed at that FAR as the signal level for which a pulse-detection probability (P_d) of some desired value is achieved (such as P_d = 95%). The FAR requirement determines the pulse-detection threshold, and the pulse-detection threshold determines the pulse-detection probability. Analyzing the FAR and the pulse-detection probability requires the statistical distribution of V_{out}, which is not adequately summarized by NEP or NEI.

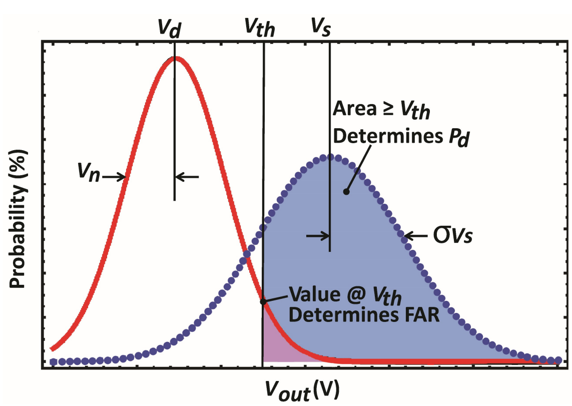

Figure 2. Statistical distributions of V_{out} for conditions with (dashed curve) and without (solid curve) the presence of an optical-signal return.

Limits in Approximating Receiver Metrics—Gaussian Distributions Do Not Suffice

It is common to approximate the probability distribution of V_{out} as a normalized Gaussian distribution (Figure 2). The mean and standard deviation characterizing the output distribution under dark conditions are respectively V_d and V_n and, when a signal is present, V_s and σ_{Vs}. Equations (5) and (6) give the receiver’s dark noise (V_n); the standard deviation of V_{out} when a signal is present can be written:

$$\sigma_{Vs} = \Omega_{TIA}\sqrt{BW \times [S_{I\,TIA} + 2\,q\,M\,F(I_d + I_b + 0.8066\,\lambda\,QE\,M\,P_s)]}\text{[V]}; \tag{9}$$

or:

$$\sigma_{Vs} = G_{TIA}\sqrt{noise_{TIA}^2 + \frac{\tau_{eff}}{q} M\,F(I_d + I_b) + N_s\,QE\,M^2\,F}\ \text{[V]}. \tag{10}$$

Equation (9) treats signal shot noise as though the peak photocurrent is DC, which is only a good approximation for rectangular signal pulses of duration longer than the APD response time. The mean photoresponse (V_s) is the sum of V_d and the voltage step (ΔV_s) given by Eqs. (1) and (2).

The Gaussian approximation is generally accurate enough to calculate the pulse-detection probability using its complementary cumulative distribution function (CCDF) evaluated at some detection threshold voltage:

$$P_d \approx \frac{1}{2}\left[1 - erf\left(\frac{V_{th} - \Delta V_s}{\sqrt{2}\,\sigma_{Vs}}\right)\right], \tag{11}$$

where V_{th} is the pulse-detection threshold, which is measured relative to V_d, allowing V_s to be replaced with ΔV_s.

The Gaussian distribution is not a sufficient approximation to calculate FAR for photoreceivers assembled from APDs characterized by larger values of the ionization rate ratio (k). APD photoreceivers are usually operated with a detection threshold set high enough to extinguish false alarms by a factor on the order of a million, such that FAR is sensitive to the detailed shape of the false-alarm-amplitude distribution many standard deviations above the mean amplitude. The true distribution of V_{out} under dark conditions is the convolution of a Poisson-weighted McIntyre distribution representing the APD multiplied dark current (P_{APD}) and a Gaussian distribution representing the input-referred noise of the amplifier (P_{TIA}):

$$P_{RX}(n_{out}) = (P_{TIA} * P_{APD})[n_{out}] \equiv \textstyle\sum_i P_{TIA}(i)\,P_{APD}(n_{out} - i) \tag{12}$$

where $P_{RX}(n_{out})$ is the probability of measuring $V_{out} = G_{TIA} \times n_{out}$, and n_{out} is the equivalent signal electron count required to achieve V_{out}.

The APD photoreceiver FAR is the probability density in time that V_{out} is at the pulse-detection threshold with a positive slope:

$$FAR = \sqrt{\frac{2\pi}{3}}\frac{V_n}{G_{TIA}} BW\,P_{RX}\left(\frac{V_{th}}{G_{TIA}}\right) = \sqrt{\frac{2\pi}{3}} n_n\,BW\,P_{RX}(n_{th}),\ \text{[Hz]} \tag{13}$$

where the conversion gain (G_{TIA}) is used to express V_n and V_{th} as equivalent input electron counts, n_n and n_{th}.

NEP and FAR both depend on the output-voltage noise, but NEP alone does not determine FAR. The dependence of FAR on the distribution of V_{out}, represented in Eq. (12) by $P_{RX}(n_{out})$, makes FAR sensitive to k and M, independent of *NEP*.

Understanding FAR and Its Relationship with Other Metrics

The relationship of FAR to k, M, and *NEP* is best demonstrated by example. For instance, three photoreceivers with the same NEP are described in Table 1, where Receiver 1 has a larger ionization-rate ratio than Receivers 2 and 3 ($k = 0.2$ *vs*. $k = 0.02$). As a result, Receiver 1 has a larger excess-noise factor ($F = 5.6$ *vs*. $F = 2.3$ at $M = 20$). To match the NEP of Receiver 1, despite lower k, Receiver 2 is operated at lower avalanche gain and Receiver 3 is assigned higher primary (unmultiplied) dark current.

The FAR for each of the receivers described in Table 1 is plotted in Figure 3 versus detection threshold characteristics. LRF sensitivity, defined as the mean signal level for which a specified pulse-detection probability and FAR are achieved, is compared in Figure 4 for FAR in the range of 1 to 150 Hz, and a pulse-detection probability of 95%. Although all three receiver configurations are characterized by the same NEP, they differ in sensitivity. Thus, NEP alone is not a good measure of LRF receiver sensitivity.

Table 1. Characteristics of Three Example LRFs Having the Same NEP

Parameter	Receiver 1	Receiver 2	Receiver 3
TIA Bandwidth (BW)	100 MHz		
TIA Input Noise ($noise_{TIA}$)	400 e-		
TIA Input Noise Spectral Intensity	1.64 x 10^{-24} A^2/Hz		
TIA Integration Time (τ_{eff})	5 ns		
TIA Conversion Gain (G_{TIA})	32 µV/e-		
TIA Transimpedance (Ω_{TIA})	1 MΩ		
APD Ionization Rate Ratio (k)	0.2	0.02	0.02
APD Gain (M)	20	13	20
APD Excess Noise Factor (F)	5.6	2.1	2.3
Primary APD Dark Current (I_d/M)	5 nA	5 nA	12 nA
APD Quantum Efficiency (QE)	80%	80%	80%
n_n	712 e-	465 e-	712 e-
NEI	44.5 photons	44.7 photons	44.5 photons
NEP (at λ=1.55 µm)	1.1 nW	1.1 nW	1.1 nW

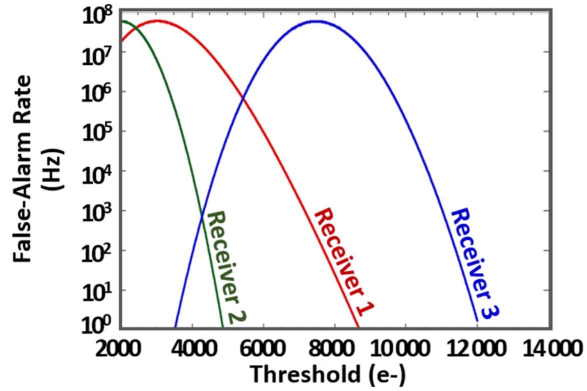

Figure 3. FAR versus detection threshold for the three receivers of Table 1, which all have the same NEP.

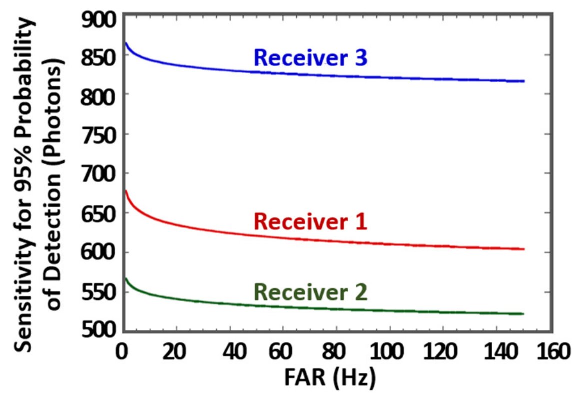

Figure 4. Sensitivity versus FAR for the three receivers of Table 1, which all have the same NEP. The sensitivity is defined as the signal level in photons at which the pulse-detection probability is 95%.

Conclusion

As demonstrated in this paper, receivers of the same NEP may differ substantially in terms of the amplitude distribution of their noise, and therefore the input-signal level required to reliably exceed the detection threshold that must be set to extinguish false alarms. Instead of optimizing dark current, gain, and excess-noise factor to minimize NEP, it is better practice to optimize for sensitivity at a FAR and detection efficiency appropriate to an application.