Relationship Between False-Alarm Rate and Probability of False Alarm

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Abstract. Discussed is the relationship between the probability of false alarm, as calculated from the false alarm rate (FAR), and the probability of detection as it relates to measuring target distance using a laser rangefinder.

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In laser rangefinding, the time-of-flight (TOF) method is used to estimate the range to a target.

In the TOF method of range estimation, a short laser pulse is transmitted and signals are back-scattered from a target. The transmitting time of a laser pulse and the reception time of the back-scattered pulse are detected by one or more photodetectors. The TOF is calculated as the time between the transmission and the reception of the pulse, and the range distance \( R \) is calculated as a function of the average speed of light given atmospheric conditions \( c \) on the basis of the relationship:

\[
R = \frac{TOF}{2} \times c.
\]

In practice, application requirements play a major role in determining the maximum range of a laser rangefinder (LRF). An LRF can provide useful measurements to a maximum distance where the probability of detecting the target \( (P_d) \) remains above some value determined by the application requirements (generally specified to be between 50% and 99%). In the absence of a specific application, generally specifying \( P_d \) is challenging because \( P_d \) depends on many target properties—range, specular and diffuse reflectance, size, orientation, and topography—as well as the intervening atmospheric properties.

When configuring an LRF, \( P_d \) is often specified relative to the probability of false alarm \( (P_{fa}) \). The \( P_{fa} \) is the probability that a false alarm will occur once per ranging event to a target at a fixed distance. In practice, for a fixed range, this is accomplished by raising the pulse-detection threshold to a level that is sufficiently high that the resulting \( P_{fa} \) is at or slightly below the required performance value. At that threshold, the optical-pulse-signal power required to exceed the voltage threshold can be measured for the specified \( P_d \).

For real-world problems, however, \( P_{fa} \) is not very useful because it is only relevant for measuring a target at one specific range—Targets are not always at the same distance, and the \( P_{fa} \) changes as a function of the duration of the time of flight to those targets at varying ranges. Moreover, the \( P_{fa} \) depends on background solar conditions and the ambient temperature, so measuring \( P_{fa} \) directly is a process that quickly becomes too unwieldy for practical application.

The benefit of specifying the performance of an LRF using a false-alarm rate (FAR) is that FAR is easily and readily measured.

To measure FAR, false alarms—resulting from electronic noise or solar background—are counted within a unit of time (i.e., false counts per second). The FAR is the count, at a given voltage threshold level, of the number of false alarms per second.

The probability of a false alarm as a function of range \( (P_{fa}) \) is then calculated as:

\[
P_{fa} = \frac{\text{FAR} \times (2R)}{c}.
\]

Similarly, the FAR can be specified for any \( P_{fa} \) requirement using:

\[
\text{FAR} = \frac{P_{fa} \times c}{(2R)}.
\]

\( P_{fa} \) equivalents are plotted as a function of target range for various FARs in Figure 1 and as a function of FAR for various target ranges in Figure 2. Similarly, FAR values are plotted as a function of target range for various \( P_{fa} \) values in in Figure 3, and as a function of \( P_{fa} \) for various target ranges in Figure 4. These plots show that achieving:

- 1% \( P_{fa} \) requires a FAR threshold of:
  - 1,000 Hz for targets at 1,500 meters; and
  - 250 Hz for targets at 5,000 meters.

- 0.1% \( P_{fa} \) requires a FAR threshold of:
  - 100 Hz for targets at 1,500 meters; and
  - 3 Hz for targets at 5,000 meters.

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Figure 1. $P_{fa}$ equivalents for various FAR values at various ranges.

Figure 2. $P_{fa}$ equivalents for various ranges for various FAR levels.

Figure 3. FAR values for various $P_{fa}$ values, specified at various ranges.

Figure 4. FAR equivalents for various $P_{fa}$ values for various ranges.